Reg. No. : 

# Question Paper Code : 31265

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2019.

Fourth Semester

Electronics and Communication Engineering

MA 2261 – PROBABILITY AND RANDOM PROCESSES

(Common to Biomedical Engineering)

(Regulation 2008)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. The moment generating function of a random variable $X$ is given by $M(t) = e^{3(e^t - 1)}$. What is $P[X = 0]$?

2. An experiment succeeds twice as often as it fails. Find the chance that in the next 4 trials, there shall be at least one success.

3. Find the marginal density functions of $X$ and $Y$ if
$$f(x,y) = \begin{cases} \frac{6}{5}(x + y^2), 0 \le x \le 1, 0 \le y \le 1 \\ 0 \ \ otherwise \end{cases}$$

4. Find the acute angle between the two lines of regression, assuming the two lines of regression.

5. Define wide sense stationary process.

6. Show that a binomial process is Markov.

7. Define power spectral density function.

8. State Wiener-Khinchine theorem.

9. Define a linear system with random output.

10. State any two properties of cross power density spectrum.

PART B — (5 × 16 = 80 marks)

11. (a) (i) If the probability density of $X$ is give by $f(x)=\begin{cases}2(1-x)\ for\ 0<x<1\\0,\ \ otherwise\end{cases}$

find its $r^{th}$ moment. Hence, evaluate $E\left[(2X+1)^2\right]$. (6)

(ii) Find MGF corresponding to the distribution $f(\theta)=\begin{cases}\frac{1}{2}e^{-\theta/2}, \theta>0\\0,\ \text{otherwise}\end{cases}$

and hence find its mean and variance. (6)

(iii) Show that for the Probability function

$p(x)=P(X-x)=\begin{cases}\frac{1}{x(x+1)}, x=1,2,3...\\0,\ otherwise\end{cases}$ $E(X)$ does not exist. (4)

Or

(b) (i) Assume that the reduction of a person's oxygen consumption during a period of Transcendental Meditation (T.M) is a continuous random variable $X$ normally distributed with mean 37.6 cc/mm and S.D 4.6 cc/min Determine the probability that during a period of T.M. a person's Oxygen consumption will be reduced by

(1) at least 44.5 cc/min

(2) utmost 35.0 cc/min

(3) anywhere from 30.0 to 40.0 cc/mm. (8)

(ii) The random variable X has exponential distribution with

$f(x)=f(X)=f(x)=\begin{cases}e^{-x},\ 0<x<\infty\\0, otherwise.\end{cases}$

Find the density function of the variable given by

(1) $Y=3X+5$

(2) $Y=X^2$. (8)

12. (a) (i) The joint PMF of two random variables $X$ and $Y$ is given by $P_{XY}(x,y)=\begin{cases}K(2x+y),\ x=1,2; y=1,2\\0,\ \text{otherwise}\end{cases}$, where $K$ is constant

(1) Find $K$

(2) Find the marginal PMFs of $X$ and $Y$. (8)

(ii) Assume that the random variable $S_n$ is the sum of 48 independent experimental values of the random variable $X$ whose PDF is given by $f_X(x) = \begin{cases} \frac{1}{3}, & 1 \le x \le 4 \\ 0, & \text{otherwise} \end{cases}$. Find the probability that $S_n$ lies in the range $108 \le S_n \le 126$. (8)

Or

(b) (i) Two random variables $X$ and $Y$ are related as $Y = 4X + 9$. Find the correlation coefficient between $X$ and $Y$. (8)

(ii) If the density function is defined by $f(x,y) = \frac{y}{(1+x)^4} e^{\frac{-y}{1+x}}$, $x \ge 0, y \ge 0$ then obtain the regression equation of $Y$ on $X$ for the distribution. (8)

13. (a) (i) If the two RVs $A_r$ and $B_r$ are uncorrelated with zero mean and $E(A_r^2) = E(B_r^2) = \sigma_r^2$, show that the process.

$$x(t) = \sum_{r=1}^{n} (A_r \cos w_r t + B_r \sin w_r t)$$ is wide-sense stationary. (8)

(ii) If $\{x(t)\}$ is a Gaussian process with $\mu(t) = 10$ and $C(t_1, t_2) = 16\, e^{-|t_1 - t_2|}$, find the probability that

(1) $X(10) \le 8$ and

(2) $|X(10) - X(6)| \le 4$. (8)

Or

(b) (i) Define Random telegraph signal process and prove that it is wide-sense stationary. (8)

(ii) Prove that sum of two independent Poisson processes is a Poisson process. (8)

14. (a) (i) Define spectral density of a stationary random process $X(t)$ Prove that for a real random process $X(t)$ the power spectral density is an even function. (8)

(ii) Two random processes $X(t)$ and $Y(t)$ are defined as follows :

$X(t) = A\cos(\omega t + \theta)$ and $Y(t) = B\sin(wt + \theta)$ where $A, B$ and $w$ are constants ; $\theta$ is a uniform random variable over $(0, 2\pi)$. Find the cross correlation function of $X(t)$ and $Y(t)$. (8)

Or

(b) (i) State and prove Wiener-Khintchine theorem. (8)

(ii) If the cross power spectral density of $X(t)$ and $Y(t)$ is $S_{XY}(w) = \begin{cases} a + \dfrac{ibw}{\alpha}; -\alpha < w < \alpha, \alpha > 0 \\ 0, \quad \text{otherwise} \end{cases}$ where $a$ and $b$ are constants. Find the cross correlation function. (8)

15. (a) (i) Prove that if the input to a time-invariant stable linear system is a wide sense process then the output also is a wide sense process. (8)

(ii) A random process $X(t)$ with $R_{XX}(\tau) = e^{-2|\tau|}$ is the input to a linear system whose impulse response is $h(t) = 2e^{-t}, t > 0$. Find the cross correlation coefficient $R_{XY}(\tau)$ between the input process $X(t)$ and output process $Y(t)$. (8)

Or

(b) (i) Let $X(t)$ be a wide sense stationary process which is the input to a linear time invariant system with unit impulse $h(t)$ and output $Y(t)$. Prove that $S_{YY}(w) = |H(w)|^2 S_{XX}(w)$ where $H(w)$ is the Fourier transform of $h(t)$. (8)

(ii) Let $Y(t) = X(t) + N(t)$ be a wide sense stationary process where $X(t)$ is the actual signal and $N(t)$ is the zero mean noise process with variance $\sigma_N^2$, and independent of $X(t)$. Find the power spectral density of $Y(t)$. (8)

———————